Topic2 – Binomial distribution

Expected value

If $X \sim B(n, p)$, that is, X is a binomially distributed random variable, with n being the total number of experiments, and p the probability of each experiment yielding a successful result, then the expected value or mean of X is:[2][3][7]

$$E[\mathrm{X}]=\mathrm{np}$$

This follows from the linearity of the expected value, along with fact that X is the sum of n identical Bernoulli random variables—each with expected value p. In other words, if $X_1, \dots, X_n$ are i.i.d. Bernoulli random variables with parameter p, then $X = X_1, \dots, X_n$ and

$$E[X] = E[X_1 + \cdots + X_n] = E[X]_1 + \cdots + E[X_n] = p + \cdots + p = np$$

Source https://en.wikipedia.org/wiki/Binomial_distribution

Mode

$$f(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\frac{f(k+1)}{f(k)} = \frac{\binom{n}{k+1} p^{k+1} (1-p)^{n-k-1}}{\binom{n}{k} p^k (1-p)^{n-k}} = \frac{(n-k)p}{(k+1)(1-p)} \; vs \; 1$$

$$\frac{(n-k)p}{(k+1)(1-p)} = 1$$

$$np - kp = k + 1 - kp - p$$

$$np + p - 1 = k$$

Let $a_k = P(X = k)$, we have

$$a_k = \binom{n}{k} p^k q^{n-k} \qquad \text{and} \qquad a_{k+1} = \binom{n}{k+1} p^{k+1} q^{n-k-1},$$

where as usual $q = 1 - p$ in binomial distribution.

We calculate the *ratio* $\frac{a_{k+1}}{a_k}$. Note that $\frac{\binom{n}{k+1}}{\binom{n}{k}}$ simplifies to $\frac{n-k}{k+1}$, and therefore

$$\frac{a_{k+1}}{a_k} = \frac{n-k}{k+1} \cdot \frac{p}{q} = \frac{n-k}{k+1} \cdot \frac{p}{1-p}.$$

From this equation we can follow:

$$\begin{aligned} k > (n+1)p - 1 &\implies a_{k+1} < a_k \\ k = (n+1)p - 1 &\implies a_{k+1} = a_k \\ k < (n+1)p - 1 &\implies a_{k+1} > a_k \end{aligned}$$

The calculation (almost) says that we have equality of two consecutive probabilities precisely if $a_{k+1} = a_k$, that is, if $k = np + p - 1$. Note that $k = np + p - 1$ implies that $np + p - 1$ is an integer.

So if $k = np + p - 1$ is not an integer, there is a single mode; and if $k = np + p - 1$ is an integer, there are two modes, at $np + p - 1$ and at $np + p$.

Not quite! We have been a little casual in our algebra. We have not paid attention to whether we might be multiplying or dividing by 0. We also have casually accepted what the algebra seems to say, without doing a reality check.

Suppose that $p = 0$. Then $np + p - 1$ is an integer, namely -1. But whatever n is, there is a single mode, namely $k = 0$. In all other situations where $np + p - 1$ is an integer, the k we have identified is non-negative.

However, suppose that $p = 1$. Again, $np + p - 1$ is an integer, and again there is no double mode. The largest a_k occurs at one place only, namely $k = n$, since $np + p$ is in this case beyond our range.

That completes the analysis when $np + p - 1$ is an integer. When it is not, the analysis is simple. There is a single mode, at $\lfloor np + p \rfloor$.

Source https://math.stackexchange.com/questions/117926/finding-mode-in-binomial-distribution/117940#117940