Topic 5. PCA

Find some orthonormal matrix *P* in *Y* = *PX*, then covariance matrix C_Y?

$$C_Y = \frac{1}{n}YY^T = \frac{1}{n}PX(PX)^T = P\frac{1}{n}XX^TP^T = PC_XP^T$$

<u>Theorem1</u>: a matrix is symmetric *if and only if* it is orthogonally diagonalizable.

=> if A is orthogonally diagonalizable, *A* is a symmetric matrix

if A is orthogonally diagonalizable => $A = EDE^T$, then

$$A^T = (EDE^T)^T = E^{TT}D^TE^T = EDE^T = A$$

A is a symmetric matrix

For a symmetric matrix *A*, let λ_1 and λ_2 be distinct eigenvalues for eigenvectors e_1 and e_2

$$\lambda_1 e_1 \cdot e_2 = (\lambda_1 e_1)^T e_2 = (Ae_1)^T e_2 = e_1^T A^T e_2 = e_1^T A e_2 = e_1^T(\lambda_2 e_2) = \lambda_2 e_1 \cdot e_2$$

Let *Y* =*PX*, then $\boldsymbol{C_Y = D}$

$C_Y = PC_XP^T$ … (1)

$C_X = EDE^T \ldots (2)$

$P = E^T \ldots (3)$

Combine (1), (2), (3)

$$C_Y = PC_XP^T = PEDE^TP^T = E^TEDE^TE = IDI = D$$