$$f(\beta_0, \beta_1) = \frac{1}{n}\left(\left(y_1 - (\beta_0 + \beta_1 x_1)\right)^2 + \cdots + \left(y_n - (\beta_0 + \beta_1 x_n)\right)^2\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)^2$$

First order condition (FOC)

$$\frac{\partial}{\partial \beta_0} = -2\frac{1}{n}\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\frac{1}{n}\sum_{i=1}^{n} y_i - \frac{1}{n}\sum_{i=1}^{n} \beta_0 - \frac{1}{n}\sum_{i=1}^{n} \beta_1 x_i = 0$$

$$\frac{1}{n}\sum_{i=1}^{n} \beta_0 = \frac{1}{n}\mathrm{n}\beta_0 = \beta_0 = \frac{1}{n}\sum_{i=1}^{n} y_i - \beta_1\frac{1}{n}\sum_{i=1}^{n} x_i = \bar{y} - \beta_1\bar{x}$$

$$\therefore \widehat{\beta_0} = \bar{y} - \beta_1\bar{x}$$

$$\frac{\partial}{\partial \beta_1} = -2\frac{1}{n}\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)x_i = 0$$

$$\frac{1}{n}\sum_{i=1}^{n} y_i x_i = \beta_0\frac{1}{n}\sum_{i=1}^{n} y_i + \beta_1\frac{1}{n}\sum_{i=}^{n} x_i^2 = (\bar{y} - \beta_1\bar{x})\frac{1}{n}\sum_{i=1}^{n} y_i + \beta_1\frac{1}{n}\sum_{i=}^{n} x_i^2$$

$$\beta_1\left(\frac{1}{n}\sum_{i=1}^{n} x_i(x_i - \bar{x})\right) = \frac{1}{n}\sum_{i=1}^{n} x_i(y_i - \bar{y})$$

$$\therefore \widehat{\beta_1} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$